A PHYSICAL THEORY OF THE FORMATION OF HOODOOS

by A. E. SCHEIDEGGER (*)

Summary — The thesis is advanced that hoodoos (mushroom-shaped erosional features in badlands) are caused by water from cloudbursts turning the corner at the brim of the overhang, flowing for a distance upside-down on the underside. This type of upside-down flow is well known as « teapot effect » in the case of tea being poured from a pot flowing down the underside of the spout rather than straight on into the cup. The measured overhang of hoodoos is in good agreement with the theoretical values obtained from hydrodynamic stability considerations.

1. *Introduction* — In semi-arid, sandy or clayey areas where there is not enough moisture and time available between cloudbursts to allow vegetation to grow profusely, the water washes gulleys and valleys into otherwise undisturbed, flat strata. Due to the lack of vegetation, the sides of the gulleys remain bare of plant growth although the more level parts show some cover with such low-lying plants as prairie grass and cactus. The whole area thus takes on a bleak appearance; the type of landscape it represents is therefore referred to as « bad lands ». Such bad lands stretch over wide areas in the interior of North America; a notable stretch of them is situated in the Red Deer River Valley of Alberta between Drumheller and Brooks.

Since erosion in bad lands does not proceed at the same pace in all localities, characteristic and sometimes fantastic features result. At times, strata are encountered which present slightly more resistance to ablation and dissolution by water than others so that « islands » are formed around which erosion takes place at a faster pace. The water now collects even more in the deeper places and the more resistive top of the developing feature acts as a protection. Thus, a series of features will eventually stand out in an area which all around has been eroded to a lower level. In general, the features thus created are pyramidal structures and are referred to as *mesas* or *buttes*. A typical array of such pyramidal structures, as photographed near East Coulee, Alberta (some 90 miles North-East of Calgary) is shown in Figure 1.

The assumption of erosion by rain explains the pyramidal structures quite naturally. However, one occasionally finds clusters of more unusual structures

(*) Dr. A. E. SCHEIDEGGER, c/o Imperial Oil Research Lab., *Calgary, Alberta,* Canada. - Presently on leave of absence from there at the Seismological Laboratory, California Institute of Technology, *Pasadena, Calif.*, U.S.A.

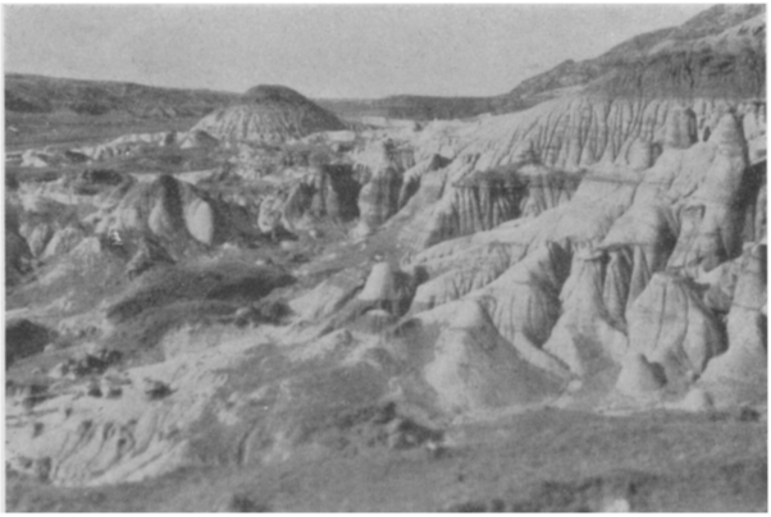

Fig. 1 - *Typical bad land topography in the Red Deer River Valley, Alberta, showing pyramidal structures.*

which have a strange, mushroom-shaped form. Instead of being pyramidal, they have an overhanging « hat » so that they have the general appearance of giant mushrooms. Such structures are called *hoodoos*. A cluster of hoodoos is shown in Figure 2.

Fig. 2 - *A cluster of hoodoos near East Coulee, Alberta.*

If one tries to assume an origin of the hoodoos which would be analogous to that of the pyramidal structures referred to above, one is at once faced with the problem as to how the water gets around the « brim » of the « hat » of the hoodoos so as to wash out their « neck ». One might think that the causes for the water turning the corner are surface forces. However, it is the writer's contention that

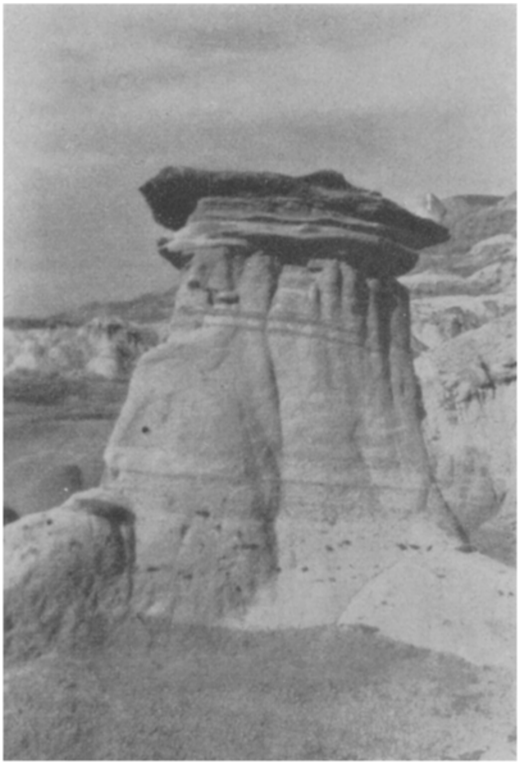

Fig. 3 - *Close-up of a hoodoo, showing its mushroom shape.*

the phenomenon is analogous to that encountered when tea being poured from a teapot runs down the underside of the spout rather than straight on into the cup. This phenomenon has been called *teapot effect*; it is not due to surface forces, interfacial tensions or such like, but is a consequence of the prevailing flow potentials.

It is the intention of this paper to establish the thesis that hoodoos are created by the teapot effect. In order to do this, we first give a more accurate description of the hoodoos, including their commonly found measurements. Then we shall describe the teapot effect, and finally analyse the bearing of the latter upon the formation of the hoodoos.

2. *Description of Hoodoos* — Let us have a closer look at the features which we have called hoodoos. A close-up photograph of such a hoodoo, taken near

East Coulee, Alberta (*) is shown in Figure 3. This particular hoodoo is 2.4 metres high, and the total overhang of the top plate is 0.3 metres. The writer measured three hoodoos of the cluster shown in Figure 2; the result of this procedure is shown in Table 1. The hoodoo shown in Figure 3 is No. 2 in Table 1.

TABLE 1. — *Measurements of Three Hoodoos.*

No.	Height	Waist	Overhang
1	3.0 m	0.6 m 1.5 m	0.4 m 0.5 m
2	2.4 m	0.8 m 1.4 m	0.3 m 0
3	1.6 m	0.8 m 1.4 m	0.2 m

In inspecting Table 1, it will be noted that in some instances, there are two numbers given. These indicate the maximum and minimum values for various cross-sections of the hoodoo in question.

The problem, then, is to explain the various overhangs of from 20-50 centimeters.

3. *Teapot Effect* — In order to proceed to our explanation of the observed overhangs on hoodoos, let us now investigate what is known about the teapot effect, as described in the Introduction. The teapot effect is not due to surface forces, but is a consequence of the prevailing flow potentials. It has been closely studied by REINER (1) and by KELLER (2).

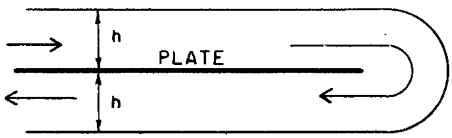

Fig. 4 - *Flow with one free surface around the edge of a semi-infinite flat plate. After* KELLER.

If we neglect gravity forces for one moment, then it can be shown that there are various possible flows when a jet of fluid leaves a nozzle with parallel walls. KELLER (2) made a study of this and came up with a variety of flows. He calculated the flow potentials for the planar case where a jet is confined between two parallel plates. The plates end, say, at $x = 0$ and the jet moves on. There are four possibilities. One is that the jet moves straight on, another that it turns around the upper as well as around the lower plate, filling the whole of space. The remaining two possibilities are where the jet turns either around the upper or around the lower plate. This is the teapot effect. In the course of his investigations, KELLER found an additional flow which has a direct bearing upon the problem of the hoodoos.

(*) Across from the picnic shelter on the Drumheller-East Coulee highway near the C.P.R. railway crossing.

Assume that there is a plate extending along the x-axis towards minus infinity, ending at $x = 0$. Then it can be shown that a free surface flow is possible whose surface has (at minus infinity) the distance $\pm h$ from the plate. The geometry of this flow is then as shown in Figure 4.

The complex potential for this flow is calculated by making a series of conformal mappings until the boundaries are of such a form that the potential can be written down easily. In order to do this, one must assume that the stream function (the imaginary part of the complex potential) is zero on both sides of the plate (since this represents *one* streamline) and that on the free surface, it is equal to a constant Q representing the total flux. The equation between the complex potential w and the complex variable z turns out to be

$$z = -\frac{4h}{\pi} \text{ lognat } \cosh \frac{\pi w}{4Q} . \tag{1}$$

One can indeed convince oneself that $w = w(z)$ satisfies, with its real and imaginary parts, the Laplace equation and that the boundary conditions as stated are also satisfied. One therefore has the required solution.

It has thus been demonstrated that, provided gravity is neglected, there exists a possible solution to the flow equations where the flow turns a corner (1). The above solution is valid only for a *thin* plate.

One still has to investigate the effect of gravity. This effect is presumably small near the corner since the hydrodynamic pressure variations are small there. However, far from the edge, the flow will be parallel to the plate on the underside. An exact solution for such a flow is:

$$\left\{ \begin{array}{l} u = \text{constant} \\ v = 0 \\ h = \text{constant} \\ p = p_0 - \rho g\,(h + y) \end{array} \right. \tag{2}$$

where u is the horizontal velocity, v the vertical velocity, p_0 the atmospheric pressure, ρ the density of the fluid and y the distance *above* the plate (so that the free surface is at $y = -h$). It is immediately obvious that Equation (2) represents an *exact* solution of the flow equations, which is possible if

$$h < p_0/(\rho g) . \tag{3}$$

It turns out, thus, that the atmospheric pressure can indeed support flow on the underside of a plate.

Equation (2) is not sufficient to « explain » hoodoos, nor the teapot effect. For, although horizontal flow on the underside of a plate can indeed exist, such flow is obviously an unstable flow: eventually, it will detach itself and drop off downward. One must therefore investigate how long the flow can follow the plate before the always-present disturbances grow sufficiently to make it detach itself. The procedure for doing this is a standard one for investigating hydrodynamic instability: small perturbations are introduced into the flow equations and their growth is analyzed. The above considerations have been applied to precisely our problem by Keller (2).

Introducing a perturbation at the edge of the plate, one can calculate the distance L at which it will have grown by the factor e. This distance does depend on the interfacial tension T between air and water. Furthermore, it depends on the form of the original perturbation. Of interest is that distance L which is the smallest in all the modes of instability that can occur. The expression for this minimum distance cannot be written down in closed form, but in two limit cases, this is possible. KELLER found:

$$\text{(4)} \qquad \text{if} \quad \frac{\rho g h^2}{T} \ll 1\,, \quad \text{then } L = \frac{2u}{g}\sqrt{\frac{T}{\rho h}}$$

$$\text{(5)} \qquad \text{if} \quad \frac{\rho g h^2}{T} \gg 1\,, \quad \text{then } L = \frac{u}{\sqrt{2}}\left(\frac{27\,T}{\rho g^3}\right)^{1/4}.$$

In hydrodynamic stability calculations it is, then, usually assumed that the flow will actually become unstable (i.e. detach itself) after it has travelled a distance of $10\,L$.

4. *Bearing of Teapot Effect on Hoodoos* — Let us now investigate the significance that the above mentioned discussion might have with regard to the formation of hoodoos.

In the case of hoodoos, the eroding agent is water. In the case of water, one has $T = 80$ dynes/cm, $\rho = 1$ g/cm^3, $g = 980$ cm/sec^2; thus

$$\text{(6)} \qquad \text{if} \quad 12\,h^2 \ll 1\,, \quad \text{then } L = u/h^{1/2} \times 0.0183$$

$$\text{(7)} \qquad \text{if} \quad 12\,h^2 \gg 1\,, \quad \text{then } L = u \times 0.0275$$

where all units are in the cgs system.

The distance L, as has been explained above, is that distance in which the most significant disturbance grows by the factor e as stated above. In hydrodynamic-stability theory, it is usually assumed that the instability will become predominant (i.e. the flow will detach itself) in a distance equal to ten times L.

It turns out that the case (b) applies if h is greater than about 1/3 cm. Then

$$\text{(8)} \qquad 10\,L \cong u \times 0.28 \text{ cm}$$

irrespective of the thickness h of the flow. It is difficult to estimate the velocity u in the flow. In a good cloudburst it will probably reach about 1-2 m/sec at the edge of the overhang. This means that the flow can continue on the underside for about 28-56 cm before detaching itself. According to earlier remarks about the mechanism of erosion, this distance of 28-56 cm is the distance by which the « hat » of the hoodoos can overhang, for, in order to erode the soft material below, the water must obviously first reach it.

It thus appears that the values postulated above from a discussion of the teapot effect are in good agreement with those actually found in the measured hoodoos. This would serve to substantiate the theory.

REFERENCES

(1) REINER M.: Physics Today, No. 9, 16-20 (1956). — (2) KELLER J. B.: J. Appl. Phys., Vol. 28, 859-864 (1957).

(*Received* 21*st November* 1958)