ESCI-555 Lecture Notes: Parameterized Convection

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1. THERMAL CONVECTION

1.1. Introduction

Imaging hitching a ride with a parcel of fluid within a body of fluid that is heated from below and cooled from above:

heating → expansion → rise → cooling → contraction → sink

The corresponding conceptual model is a layer of fluid heated from below and cooled from above, so that there is a temperature difference of $\Delta T$ across the layer and at steady state the heat conducted across the bottom and top boundaries is equal and given by the heat flux $q$. The fundamental concept is that convection is characterized by temperature fluctuations at the macroscopic scale.

![Conceptual model for Rayleigh-Benard convection. A fluid layer of thickness $H$ is heated from below and cooled from above, by maintaining a temperature difference across the layer of $\Delta T$. At steady state the heat flux across the bottom and top boundaries, $q$, is the same. A spherical blobs shown in red (blue) of hot (cold) and buoyantly rising (sinking) indicates the convective motions of the fluid. The cyan-colored circles with arrows indicate the overall convective motions after the onset of convection. The length scales over which convective velocities change is $H$, as is the length scale of temperature changes that scale as $\Delta T$. The temperature profile shown in orange indicates the formation of hot and cold thermal boundary layers.](image)

The following analysis pertains to conditions where the viscous force is dominant over inertial forces, as is the case for Earth’s mantle. In this case the viscous force is balanced by the buoyancy force

$$|\mu \nabla^2 \mathbf{u}| \sim |g \rho \gamma \Delta T|,$$

or equivalently

$$|\nu \nabla^2 \mathbf{u}| \sim |g \gamma \Delta T|.$$

Here $\mu$ is the dynamic viscosity, $\nu = \mu/\rho$ is the kinematic viscosity, $\rho$ is the density of the fluid, $\mathbf{u}$ is the fluid velocity, $g$ is the acceleration due to gravity, $\gamma$ is the thermal expansivity, $T$ is temperature and $\Delta T$ is the difference in temperature between bottom and top of the fluid layer.

Using $H$ as the characteristic length scale, and $U$ as the characteristic velocity scale, we can non-dimensionalize the force balance to obtain

$$U \sim \frac{g \gamma \Delta T H^2}{\nu}. $$
The characteristic time scale for buoyant rise across the layer is given by

$$\tau_U \sim H/U.$$  

As hot or cold fluid traverses the fluid layer, it loses heat to the surrounding fluid by conduction. For convection to occur temperature variations of the order of $\Delta T$ have to persist across the entire thickness of the layer, we define the characteristic diffusion time scale, $\tau_\alpha$, using the layer thickness, $H$, and the thermal diffusivity, $\alpha$, to obtain

$$\tau_\alpha \sim H^2/\alpha.$$  

For convective motions to occur $\tau_\alpha \gg \tau_U$, or

$$1 \ll \frac{\tau_\alpha}{\tau_U} = \frac{g\gamma \Delta TH^3}{\alpha \nu} \equiv Ra,$$

where the term on the right hand side of this equation is called the Rayleigh number. It is found that if Ra is greater than some critical value, Ra_c $\approx$ 1000, the fluid layer will convect.

**SUMMARY POINTS**

1. At small Rayleigh numbers (but above the critical Ra), convection in a layer of uniform thickness takes the form of rolls and hexagonal cells.
2. At high Rayleigh numbers heat is conducted across (thin) thermal boundary layers at the heated bottom and/or cooled top. The thermal boundary are in a perpetual state of critical instability, giving rise to thermally buoyant plumes.
3. During high Ra convection, heat is advected by plumes across the interior and the interior temperature is on average uniform (adiabatic).
4. If viscosity is temperature dependent it affects the cold thermal boundary layer, which can become a sluggish lid or a stagnant lid.
   - At viscosity contrasts of <100 the upper boundary is entirely mobile.
   - At viscosity contrasts of $10^2$ to $10^3$ the upper boundary becomes sluggish (sluggish lid).
   - At viscosity contrasts of $10^4$ the upper boundary becomes stagnant (stagnant lid).
5. If viscosity is temperature(volatile)-dependent there is a feedback that leads to self-regulation: temperature/volatiles increase $\to$ viscosity decreases $\to$ Ra increases & heat flow/degassing increase $\to$ temperature/volatiles decreases.
Figure 2
Two-dimensional rolls and three-dimensional hexagonal cells in a fluid layer heated from below, just above critical. (Bejan, 1995, Figure 5.22)

Figure 3
Isothermal convection, heated from below and cooled from above at high Rayleigh number. (Bejan, 1995, Figure 5.23)
The viscosity of corn syrup is approximately an Arrhenius function of temperature, and it is necessary to use fairly high values of the Prandtl number, $Pr = \frac{\nu c_p}{k}$, where $\nu$ is the viscosity, $c_p$ is the specific heat capacity, and $k$ is the thermal conductivity. Previous studies have shown that $Pr < 10$ is a good approximation for many geophysical flows. In our experiments, we have a range of Prandtl numbers between 4 and 10, which is typical for high-Prandtl-number fluids. The Prandtl number for water is approximately $Pr \approx 6.2$. We use tank depths of 10, 17, 20, and 25 cm, with aspect ratios ranging from 1 to 3. The corn syrup we use has a viscosity of $\nu = 10^{-3}$ Pa s, and the overall viscosity of the fluid layer is approximately $\nu = 10^{-2}$ Pa s. The density of the fluid layer, $\rho$, varies with temperature, and we normalize with respect to the temperature at the top, $T_0$, and the bottom, $T_1$. Hereafter, we will refer to dimensionless temperatures, $T^*$, defined as $T^* = (T - T_0)/(T_1 - T_0)$, where $T$ is the temperature of interest. Normalized velocities, $V^*$, are similarly defined as $V^* = V/H$. The boundary layer in the downstream direction is thinned to subcritical lateral growth. The rightmost quarter of the area of Fig. 3 ((note the thickened TBL at the base of the conduit).}

**Figure 4**

Figure 1 of Manga and Weeraratne, *Physics of Fluids*, 1999. Experimental apparatus.

**Figure 5**

Figure 2 of Gonnermann et al., *Earth and Planetary Science Letters*, 2004. Shadowgraph image of convection with temperature-dependent viscosity at Ra $\sim 10^9$. Height approximately 0.5 m.
flows with no-slip boundaries, the thermal boundary layer thickness scales with Pe$^{1/3}$, e.g., Ref. 15, for all experiments except those of Zhang et al. The solid curve is given by Eq. 17.

*Dimensionless number* is the vertical position, $z/D$.

$\theta$ is temperature, and $T$ is temperature. Figure 3 shows $T$ as a function of a suitably defined $Ra$, with $Ra$ varies by about $2/3$ as a function of $Ra$. The solid line is the dimensionless function of the Rayleigh number based on properties of the lower thermal boundary layer. The solid line is given by Eq. 17.

For the case of variable-viscosity convection, $Ra$ should be for free-slip surfaces. The thermal boundary layer arguments, used to derive Eqs. 17. The period of thermal formation, $T$, is shown. Re is between about 1 and 100. Thus fluid inertia does not affect thermal passage of hot thermals. Measurements are made once every 20 s for the case of variable-viscosity convection. Two additional features of the results in Fig. 2 are of interest. First, the data in Fig. 2 have no dependence on the Rayleigh number for $Re > 100$. Second, $Re$ as a function of $Ra$ for published experimental data. See also Ref. 14.

We confirm visually that thermals did indeed form in all experiments. The period of hot thermal formation, $T$, is determined from spectral analysis of data. Details of the experimental setup, procedure, and data are published in a thesis.

FIG. 1. Vertical temperature distribution; $\delta$ denotes boundary layer thicknesses, $z$ is the vertical position, $T$ is temperature, and $\theta$ is the dimensionless temperature.

Figure 6

Figure 1 of Manga et al., *Physics of Fluids*, 2001. Vertical temperature distribution during stagnant lid convection.
Fig. 1. Temperature fluctuations at three or four middle thermocouples under conditions of thermal equilibrium. (a) Unsteady convection dominated by large scale flow $R_{\alpha_2} = 2 \times 10^5$. (b) Increased plume activity along with large-scale flow $R_{\alpha_2} = 4.7 \times 10^6$. (c) Plume-dominated convection with short period fluctuations and a constant average temperature $R_{\alpha_2} = 5.5 \times 10^7$. Dimensionless temperature, $O_T$, is defined as.

We attribute the short-period temperature fluctuations in Fig. 1 to rising and sinking thermal plumes. We refer to flows in which we only observed short-period temperature fluctuations, e.g. Fig. 1c, as 'plume-dominated' convection. Indeed, in this limit we observe rising and sinking mushroom-shaped plumes, as shown in Fig. 2. These plumes always appear to consist of a plume head and tail. The detached plume heads observed by Yuen et al. [12] in 2D numerical calculations with free-slip boundaries may be due to the presence of a large-scale flow [13] and do not occur in 3D calculations [14]. In Fig. 1d, we summarize the conditions at which we observe each of the three convective styles: steady, unsteady, and plume-dominated.

In Fig. 3a, we show histograms of the temperature fluctuations corresponding to Fig. 1a–c. For the flows that we call 'unsteady', see Fig. 1d, we observe a broad distribution with a superimposed peak at $O_T \approx 0.7$ that is due to plumes. Here, $O_T = \frac{T}{T_0}$. For plume-dominated flows, we observe a narrow distribution that is approximately exponential. Experimental studies with low Prandtl number fluids (very high Reynolds numbers), find an abrupt change in the distribution of temperature fluctuations, from a Gaussian to an exponential distribution, that defines the transition to 'hard thermal turbulence' [15,16]. Our distributions change gradually with increasing...
Figure 8

Figure 12 of Manga and Weeraratne, *Physics of Fluids*, 1999. Nu-Ra.
1.2. Conductive heat transfer

A FEW THINGS TO REMEMBER

1. Energy has units of $J = N\cdot m = kg\cdot m^2/s^2$.
2. Power has units of $W = J/s = N\cdot m/s = kg\cdot m^2/s^3$.
3. Heat flux, $q$, has units of $W/m^2$.
4. The specific heat capacity, $c_p$, has units of $J/(K\cdot kg)$.
5. Thermal conductivity, $k_h$, has units of $W/(m\cdot K)$.
6. Thermal diffusivity, $\alpha = k_h/(\rho c_p)$, has units of $m^2/s$.

Fourier’s law of heat conduction in one dimension

$$q = -k \frac{dT}{dz}$$  

or more generally

$$q = -k \nabla T,$$

where $k$ is thermal conductivity in units of $W\cdot m^{-1} K^{-1}$, $T$ is temperature and $q$ is heat flux.

- What are the units of $q$?

From energy balance of a representative elemental volume it is possible to derive an equation for the conservation of energy for a solid or motionless fluid

$$\rho c_p \frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{q}.$$  

where $\rho$ is density and $c_p$ is heat capacity. For constant material properties

$$\rho c_p \frac{\partial T}{\partial t} = k \nabla^2 T$$  

or

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T,$$

where $\alpha$ is the thermal diffusivity ($m^2 s^{-1}$).

1.3. Nusselt-Rayleigh number from assumption of critically unstable thermal boundary layer

It should be noted that alternate derivations of the Nu-Ra scaling relation exist that result in relationships of $Nu \sim Ra^{1/3}$. In fact, experiments show that measured convective heat fluxes fall somewhere between $Nu \sim Ra^{1/4}$ and $Nu \sim Ra^{1/3}$, depending on the magnitude of $Ra$, with the majority of experiments falling closer to $Ra^{1/3}$ than $Ra^{1/4}$. A very simple analysis is based on the following consideration.

A layer of fluid of thickness $H$, which is heated from below and cooled above begins to convect at $Ra \sim 10^3$. At $Ra > 10^3$, the layer convect in the form of two-dimensional rolls or hexagonal cells called Bénard cells. At $Ra \gg 10^3$, this orderly convection pattern breaks down and the flow is by thermals or plumes. In this case the spatially averaged temperature...
in the interior of the layer is approximately uniform, with a cold and hot thermal boundary later (TBL) at the top and bottom, respectively. The TBL remains in a critical state, that is it thickens to the point of instability, where thermals or plumes are generated. This critical thickness, $\delta_T$, corresponds to the critical Rayleigh number, that is $Ra_{\delta_T} \approx 10^3$, where

$$Ra_{\delta_T} \equiv \frac{g \gamma \Delta T \delta_T^3}{\alpha \nu}.$$  

We can thus express the Rayleigh number for the layer in terms of $Ra_{\delta_T}$ as

$$Ra_H = Ra_{\delta_T} \left( \frac{H}{\delta_T} \right)^3 \sim 10^3 \left( \frac{H}{\delta_T} \right)^3$$  

and, hence,

$$\delta_T \sim 10 H Ra_{\mu}^{-1/3}.$$  

Because heat into and out of the convecting layer is by diffusion across the TBL, the actual heat flux is

$$q_{\text{convective}} \sim k \frac{\Delta T}{\delta_T},$$  

whereas the conductive heat flux in the absence of convection would be

$$q_{\text{conductive}} \sim k \frac{\Delta T}{H}.$$  

From this we obtain

$$Nu \equiv \frac{q_{\text{convective}}}{q_{\text{conductive}}} \sim \left( k \frac{\Delta T}{\delta_T} \right) / \left( k \frac{\Delta T}{H} \right) \sim \left( k \frac{\Delta T}{10 H Ra_{\mu}^{-1/3}} \right) / \left( k \frac{\Delta T}{H} \right)$$  

or

$$Nu \approx 0.1 Ra_{H}^{1/3}.$$  

This result is rather remarkable, because it indicates that the thermal boundary layers are decoupled from the one another or the interior of the convecting layer. As $H$ is increased conductive heat flow decreases, whereas convective heat flow remains constant. That this is so, can be seen by solving the $Nu \sim Ra^{1/3}$ relation for $q_{\text{convective}}$

$$q_{\text{convective}} \sim k \Delta T \left( \frac{g \gamma \Delta T}{\alpha \nu} \right)^{1/3}.$$  

Clearly, there is no dependence of $q_{\text{convective}}$ on $H$. It is, however, important to note that $Nu \sim Ra^{1/3}$ is a limit, and under a wide range of realistic conditions, especially at finite Pr where inertial forces may perhaps not be entirely negligible, it is found

$$Nu = a Ra^b, \quad \text{where} \quad \frac{1}{4} \leq b \leq \frac{1}{2}.$$  

1.4. Advanced considerations for Nusselt-Rayleigh number

As you can see, there seems to be some discrepancy in relationship between Nusselt number and Rayleigh number. Based on a large amount of experimental work, it turns out that there is no single relation that fits the Nusselt number for all values of Rayleigh number. Overall, it has been found that

$$Nu = a Ra^b.$$  

10 ESCI-555
For example, the analysis from Section refsec:Nu-Ra-TurcotteSchubert gives
\[ a = Ra^{-b} \quad \text{and} \quad b = 1/3, \]
where the critical Rayleigh number is \( Ra_c \approx 1000 \). However, in reality constants \( a \) and \( b \) depend on the value of \( Ra \) and also on \( Pr \). For example, Schubert et al. (Mantle Convection in the Earth and Planets, 2000) suggest that at very large \( Pr \) and \( Ra = 10^6 \)
\[ a = 0.27 \quad \text{and} \quad b = 0.3185 \]
are good values. A recent comprehensive analysis by Grossmann and Lohse (J. Fluid Mech., 2000) finds that across a wide range of \( Ra \) and \( Pr \) the following relationship is a reasonable approximation
\[ Nu = 0.27Ra^{1/4} + 0.038Ra^{1/3}. \]

Unfortunately, for earth scientists dealing with sub- and/or super-solidus convection in materials with a temperature dependent viscosity, the matter becomes more complicated. The thickness of the hot and cold thermal boundary layers is no longer equal and the average interior temperature is shifted to values somewhat greater than \( (T_{cold} + T_{hot})/2 \). Furthermore, it becomes more tricky to define the Rayleigh number, because viscosity is no longer a constant. One way this has been approached is outlined in Manga and Weeraratne (Physics of Fluids, 1999), where they defined the Rayleigh number based on the viscosity at \( T = (T_{cold} + T_{hot})/2 \), and obtained a Nu-Ra relationship of
\[ Nu = 1.46 \left( \frac{Ra}{Ra_c} \right)^{0.281} \]
for \( Ra \lesssim 10^6 \) and provided that \( Pr \) is sufficiently large.

1.5. Parameterized convection

The Nu-Ra scaling pertains to the steady heat transfer across a layer of fluid that is heated from below and cooled from above, by holding the temperatures at the top and bottom constant. Here we assume that this scaling applies to a convecting layer that is cooling, at a rate that is sufficiently slow, so that any given time the heat transfer from the layer can be obtained from the Nu-Ra relationship.

The rate of heat transfer out of a convecting layer of thickness \( H \) is
\[ \rho c_p HA \frac{dT}{dt} = q_{\text{convective}} A, \]
where \( A \) is surface area. Using Nu-Ra, we know that
\[ q_{\text{convective}} = a Ra^b \quad q_{\text{conductive}} = \frac{a k \Delta T}{H} Ra^b. \]
Therefore,
\[ \frac{dT}{dt} = \frac{a a \Delta T}{H^2} Ra^b. \]
In the presence of internal heat generation \( Q \) (units of W m\(^{-3}\)) this equation becomes
\[ \frac{dT}{dt} = \frac{a a \Delta T}{H^2} Ra^b + \frac{Q}{\rho c_p}. \]
For instantaneous heating or cooling of a half-space Turcotte & Schubert define thickness of the thermal boundary layer as that thickness over which the temperature changes by 90% of the temperature difference between the surface and the far field interior. Solving the heat equation using the similarity variable

$$\eta = \frac{z}{2\sqrt{\alpha t}},$$

where $z$ is the spatial variable and $\sqrt{\alpha t}$ is the characteristic diffusion distance, gives for the heat flux at the surface

$$q = \frac{k\Delta T}{2\sqrt{\pi \alpha t}},$$

where the factor of 1/2 accounts for the fact that the temperature difference across the upper thermal boundary layer (the lithosphere) is $-\Delta T/2$. Assuming that the oceanic lithosphere is equivalent to a thermal boundary layer, Turcotte & Schubert equate $L$ to the thickness of the lithosphere as it cools with time, $t$, over which it has moved a distance $X = U_s t$ from the spreading ridge. Here $U_s$ is the spreading rate. The average residence time of oceanic lithosphere on the Earth’s surface, $\tau$, that is the average time before oceanic lithosphere subducts, is given by the area of the ocean floor $A_o = A_E - A_c$, the total length of ridge, $Y_r$ and the average spreading rate, $U_s$. We thus have

$$\tau = \frac{A_o}{Y_r U_s}.$$

Therefore the average heat flux at the surface of the oceanic lithosphere, $\bar{q}$ can be obtained from integration of Equation 31. as

$$\bar{q} = \frac{k\Delta T}{\sqrt{\pi \alpha \tau}}.$$

Substituting for $\tau$ gives

$$\bar{q} = \frac{k\Delta T\sqrt{U_s}}{\sqrt{\pi \alpha A_o/Y_r}}.$$

Within the framework of our convective model, the convective heat flux $q_{\text{convective}}$ equals the heat conducted through the upper thermal boundary layer (the oceanic lithosphere), that is

$$q_{\text{convective}} = \bar{q} = \frac{k\Delta T}{2L}.$$

Equating $\bar{q}$ with $q_{\text{convective}}$ and solving for the spreading rate thus gives

$$U_s = \frac{\pi \alpha A_o}{Y_r} \frac{1}{(k\Delta T)^2} q_{\text{convective}},$$

which is the same as Equation 45 of Tajika and Matsui (1992), except for the factor of 2 squared, due to the aforementioned definition of the temperature difference across the lithosphere.

**TAKE-HOME MESSAGE**

1. Using Nu-Ra one can calculate mantle temperature as a function of time.
2. This requires appropriate definition of the Ra and exponent $b$ to account for internal heating and temperature/volatile-dependent viscosity and other 'complicating' factors.

3. Using Nu-Ra one can also calculate the MOR spreading rate, $U_r$.

4. In addition, Nu-Ra gives the depth of melting, $d_m$, beneath MORs, via mantle temperature (Section 3).

5. $U_r$ together with $d_m$ allow calculation of the melt production rate, which in turn is used to calculate the mantle degassing rate.

1.6. Internal heating

Figure 8.3 of Davies.
2. MANTLE CONVECTION

2.1. Plate-scale and layered convection

Figure 2: The subduction connection. a, Models of noble-gas evolution have classically implied that only the upper mantle is effectively degassed\(^1\), with the 660-kilometre seismic discontinuity producing a layered mantle by representing a boundary to the subduction of plates and the flow of material into the deep mantle. b, Seismological images\(^2\), however, now provide strong evidence that plates can penetrate into the lower mantle, with the associated counterflow producing whole-mantle convection. The model of Gonnermann and Mukhopadhyay\(^3\) shows that, contrary to many expectations, this mode of whole-mantle convection is quite compatible with observations of helium isotopes and other noble gases. Graphic not to scale.

SUMMARY POINTS: PLATE-SCALE FLOW

1. The Earth has rigid plates, which in essence represent a cold, high-viscosity thermal boundary layer.
2. Plates can break and thereby move relative to each other and subduct. This is not the case for other planets, such as Mars or Venus.
3. When plates sink, they drive flow in the mantle at the spatial scale of plate tectonics. This length scale is larger than it would in the absence of plates.
4. Conservation of mass requires that the descending flow is balanced by (passive) ascending flow, some of which ultimately reaches the Earth’s surface at mid-ocean ridges.
5. Plate motion is a consequence of negative buoyancy of the oceanic lithosphere. Plates are pulled at subduction zones (plate pull) and plates are pushed at ridges (ridge push). Which is the dominant force is somewhat controversial. These processes are reflected in the topography of oceanic lithosphere.
6. The presence of continental plates that cannot sink into the mantle complicates plate-scale convection.
7. The presence of phase transitions in the transition zone modulates plate-scale flow.
8. Increase in viscosity with depth in the lower mantle also modulates plate-scale flow.

SUMMARY POINTS: LAYERED MANTLE CONVECTION

1. Viscosity is thought to increase by a factor of 10-100 from upper to lower mantle. This result comes from the analysis of geoid anomalies.
2. Ocean-island basalts (OIBs) are isotopically distinct from mid-ocean ridge basalts (MORBs). MORBs are produced by melting of upper mantle. The mantle source of OIBs remains controversial, but they typically thought to originate in the lower mantle. This implies that there is considerable geochemical differences between upper mantle and OIB mantle reservoirs.
3. The transition zone can “retard” buoyant flow across it.
4. Small compositional changes (e.g., Fe, Si) between upper and lower mantle may translate to compositional density differences and convective stratification.
5. The combination of an increase in viscosity, transition zone, perhaps compositional density changes in conjunction with observed geochemical differences between OIBs and MORBs has led to the hypothesis of layered mantle convection.
6. Most numerical modeling of mantle convection suggests that layered mantle convection, if it exists at all, is “weak”.
7. Seismic tomography indicates that slabs penetrate into the lower mantle, albeit not completely unfettered by the transition zone.
8. Alternative hypotheses for geochemical mantle heterogeneity are (1) that the mantle is heterogeneous at a small scale and that MORBs and OIBs are produced from different parts of the same heterogeneous mantle assemblage; (2) that OIBs originate from some ancient (primordial) layer, perhaps D”, or some "stealth" mantle layer.

SOME COMPLICATING ISSUES

1. Mantle viscosity is temperature dependent.
2. Mantle viscosity depends on water content.
3. Internal heating affects Nu-Ra scaling.
4. Temperature-dependent viscosity can result in plume heads with persistent tails.
5. Mobile upper boundary vs. sluggish lid vs. stagnant lid.

SOME QUESTIONS

1. If slabs penetrate into the lower mantle mass balance requires that there is a return flow from lower mantle to upper mantle. Will this have led over Earth’s history to complete homogenization of upper and lower mantle?
2. Where do OIBs originate?

Figure 10.1. Numerical convection sequences. Left: with temperature-dependent viscosity (maximum viscosity 100 times the ambient viscosity). Right: the same with low-viscosity weak zones in the thermal boundary layer. The stiffness of the boundary layer inhibits the flow in the left sequence. The weak zones on the right allow pieces of the boundary layer to move more readily, simulating the motion of lithospheric plates. (Full technical specifications of this and subsequent numerical models are given in Appendix 2.)
Figure 10.3 of Davies. Higher viscosity in the lower mantle decouples upper and lower mantle flow.
Figure 10.6 of Davies. Constant viscosity downwellings do not penetrate the transition zone, but instead pile up until sufficient thermally dense mass has accumulated to cause “overturn”.

Figure 10.6. Constant viscosity convection sequence in which phase transformation buoyancy causes temporary layering of the flow. The buoyancy corresponds to a Clapeyron slope of $-3$ MPa/K. From [17]. Copyright by Elsevier Science. Reprinted with permission.
Figure 14

Figure 10.7 of Davies. If cold downwellings are of high viscosity, then they penetrate the transitions zone. This is even more accentuated in three-dimensions.
Figure 10.13. Frames from a series of convection models like that of Figure 10.12, with different variations of viscosity with depth. The cases are labelled with the magnitude of the viscosity step at 700 km and the magnitude of the superimposed smooth exponential increase, if any. The top left frame has no depth dependence. End boundary conditions are periodic, except in the lower right frame, which has no-flow ('mirror') end walls.

Figure 15
Figure 10.13 of Davies. Depth-dependent increase in viscosity causes plates to pile up and spread out in the lower mantle.
Figure 2. Series of mantle cross-sections through the recent P-wave model of Kárason & van der Hilst (2000) to illustrate the structural complexity in the upper-mantle transition zone and the regional variation in the fate of the slabs. Dashed lines are drawn at depths of 410, 660 and 1700 km, respectively. The model is based on short-period, routinely processed P, pP and PKP travel-time residuals (Engdahl et al. 1998) and a large number of PP-P and PKP-P\textsubscript{diff} differential times measured by waveform cross-correlation from long-period seismograms. The global model was parametrized with an irregular grid of constant-wave-speed cells, which allows high resolution in regions of dense data coverage, and three-dimensional finite frequency sensitivity kernels were used to account for different periods at which the measurements were made. With this technique, the low-frequency data can constrain long-wavelength mantle structure without preventing the short-period data from resolving small-scale heterogeneity.

Figure 16
Figure 2 of Albarede and van der Hilst (2002).
2.2. Plume-scale convection

Figure 1 | Isotope ratios and Earth’s mantle. a, As oceanic plates are pulled apart at mid-ocean ridges, the upper mantle rises in their place and (partially) melts. Uranium, thorium and helium in this portion of mantle are transferred to the magma, which migrates to the surface to form crust. Helium is lost during crystallization of the melts, but uranium and thorium are retained in the crust and are ultimately returned to the mantle by plate subduction (Fig. 2). Thus, the upper mantle becomes ‘degassed’ and the \((U+Th)/He\) ratio increases, which with time translates into higher \(^4\text{He}/^3\text{He}\) ratios. b, By contrast, ocean islands show a low \(^4\text{He}/^3\text{He}\) ratio, thought to reflect a deep-mantle source of underlying mantle plumes. Graphic not to scale.

Figure 17

Figure 1 of Elliott, Nature, 2009.
Figure 11.5. Thermal plumes in laboratory experiments, formed by injecting hot or cold dyed fluid into otherwise identical fluid. The fluid has a strong temperature dependence of viscosity. (a) The buoyant fluid is hot, and the plume viscosity is about 1/300 times that of the surrounding fluid. A spiral structure forms in the head due to thermal entrainment of ambient fluid. From Griffiths and Campbell [24]. (b) The injected fluid is cooler and hence denser and more viscous than the ambient fluid. There is little entrainment of cooled surrounding fluid, and only a very small head forms. From Campbell and Griffiths [25]. Copyright by Elsevier Science. Reprinted with permission.
McNutt, 1998], to the topography and geometry of the global mid-ocean ridge system [Morgan et al., 1987; Parmentier and Morgan, 1990; Small, 1995; Abelson and Agnon, 2001; Ito et al., 2003] and to cause lithospheric instabilities leading to continental breakup [e.g., Morgan, 1974; White and McKenzie, 1989; Arndt and Christensen, 1992]. Viewing mantle plumes as an intrinsic part of how the Earth cools led also to efforts toward self-consistent models for the thermal and compositional evolution of the planet as a whole [e.g., Christensen, 1984a; Zindler and Hart, 1986; Turcotte and Kellogg, 1986; Kellogg and Wasserburg, 1990; Davies and Richards, 1992; Davies, 1990, 1993; Campbell and Griffiths, 1992, 1993; Christensen and Hofmann, 1994; O’Nions and Tolstikhin, 1996; Albarede, 1998; Tackley, 1998a, 1998b; Coltice and Ricard, 2002; van Keken and Ballentine, 1999; Coltice et al., 2000; Tackley and Xie, 2002; Korenaga, 2003; Samuel and Farnetani, 2003]. The plume–hot spot model is also useful for comparative planetology. Basic differences in the nature of volcanism on the Earth, where most volcanism occurs at plate boundaries, compared with Venus and Mars, both of which have hot spot–like features but no plate tectonics, likely reflect fundamental differences in the way these terrestrial planets have evolved thermally and compositionally.

[3] Perhaps because of the riveting appearance of hot spot tracks on the Earth’s surface (Figure 1), the apparent fixity of hot spots relative to moving plates has historically captured the greatest attention and been often considered a defining characteristic of such features. In recent years, however, an improved resolution of hot spot motion has challenged, or at least redefined, the notion of hot spot fixity. For example, a number of paleomagnetic studies have shown that hot spots may be divided into at least two global families that move relative to each other at a rate currently less than about 1 cm yr\(^{-1}\): the Pacific family and the Indo-Atlantic family, which may or may not include Iceland [e.g., Raymond et al., 2000; Norton, 2000; Courtillot et al., 2003]. Within each group, however, there is negligible relative motion between individual hot spots [e.g., Burke et al., 1973; Molnar and Atwater, 1973]. Furthermore, building on earlier work by Gordon and Cape [1981] and Morgan [1981], Tarduno et al. [2003] find that the paleolatitude of the Hawaiian hot spot is not fixed. Consequently, these authors argue that the well-known bend in the Hawaiian-Emperor seamount chain, and possibly that in the Louisville hot spot track, may be dominated by motion of the underlying plumes relative to the Pacific plate rather than the reverse, which is the conventional explanation. The movement of the hot spots is thus interpreted to reflect the influence of lower mantle flow on the plume source at the base of the mantle [e.g., Steinberger, 2000; Steinberger and O'Connell, 1998, 2000; Steinberger et al., 2004]. Although it is a contentious study [Gordon et al., 2004], the conclusion of Tarduno et al. [2003] that hot spots move relative to overlying plates is not surprising. Mantle convection and stirring is expected to be time-dependent, and plumes that originate at the base of the mantle should drift.

Figure 1. A map of hot spots that have both well-defined hot spot tracks and flood basalts at their origins [from Duncan and Richards, 1991].

Figure 19
Figure 11-13 of Davies.
causes thickening and thinning of the thermal boundary layer beneath the central upwelling and downwellings at the sidewalls, respectively. Plumes move along the lower boundary toward the central upwelling at a rate determined by the horizontal velocity at the top of the thermal boundary layer, while ascending plume heads and conduits are drawn towards the central upwelling at a rate governed by velocities in the fluid interior. At large velocity ratios ($V \gg 10$), and depending on $V$, plume formation is increasingly suppressed over the distance between the sidewalls and the central upwelling. Plume suppression occurs either because the thermal boundary layer is insufficiently thick to form plumes, or because nascent plume instabilities are advected into the central upwelling before they can grow into plumes. In the extreme case of $V \approx 10$ and $V \approx 100$, the thermal boundary layer appears to be insufficiently thick to form plumes and all of the boundary layer fluid is advected horizontally into the central upwelling, where it forms a buoyant sheet. We note that the suppression of plume-driven flow is not simply due to a lower $Ra$ - the flow is plume-dominated in experiments conducted in the absence of imposed large-scale stirring.

The viscosity ratio $V$ has three effects on the flow. First, for a given temperature difference $v_T$, the size and rise velocity of plume heads is determined by the boundary layer thickness, which depends on the viscosity of the interior fluid.
across the convecting system. Two-dimensional calculations cannot accurately capture the axisymmetric head-tail structure of Earth-like mantle plumes. Figure 5c is a shadowgraph image from a laboratory experiment on convection in a temperature-dependent fluid in which the cold stagnant lid is mechanically stirred into the underlying layer using a conveyor belt. The resulting strong cooling of the interior leads to axisymmetric plumes with large heads and narrow trailing conduits.

3. COMPOSITION, STRUCTURE, AND PHYSICAL PROPERTIES OF THE PLUME SOURCE WITHIN D00

The results from a number of laboratory studies of transient high $Ra$ convection indicate that even when the flow is composed of low-viscosity upwellings with head-tail structures, complicated and destructive interactions between such plumes lead to their being spatially and temporally unstable and short-lived (Figure 6) [e.g., Jellinek et al., 1999; Lithgow-Bertelloni et al., 2001; Jellinek et al., 2003]. That is, the presence of large viscosity variations is a necessary but insufficient condition for long-lived Earth-like mantle plumes. We have argued that large viscosity variations arise from plate tectonics and thus that these conditions are a consequence of the upper boundary condition for mantle convection in the Earth. An additional natural question to consider is whether plume stability is a consequence of the physical state of the plume source region, that is, the lower boundary condition for mantle convection. [29]

Over the last 5 decades, explanations for D00, which constitutes the lowermost 200 km of the mantle, have evolved in complexity in response to a growing number of inferred constraints derived from seismological, geodynamic, geomagnetic, and geochemical studies. The original conceptual picture of D00 as essentially a global phase boundary [cf. Bullen, 1949, 1950] has expanded to variously include (Figure 7) a thermal boundary layer [cf. Stacey and Loper, 1983] locally modulated by subduction [e.g., Houard and Nataf, 1993; Nataf and Houard, 1993; Sidorin and Gurnis, 1998], chemical components left over from the formation and early differentiation of the planet (e.g., see discussions by Ringwood[1975] and Anderson[1989]), a "slab graveyard" [e.g., Dickinson and Luth, 1971; Chase, 1981; Hofmann and White, 1982; Ringwood, 1982; Davies and Gurnis, 1986], chemical "dregs" segregated from subducted lithosphere [e.g., Chase, 1981; Hofmann and White, 1982; Zindler et al., 1982; Christensen and Hofmann, 1994; Marcantonio et al., 1995], partial melt [e.g., Williams and Garnero, 1996], and metals from the outer core [e.g., Knittle...}

**Figure 7.** Schematic illustration of several models for D00. Within the context of plate tectonics, D00 has been explained variously as (a) a phase change, (b) a thermal boundary layer, (c) a compositional boundary layer, (d) ponded chemical dregs from subducted lithosphere, and (e) a slab graveyard.
3. MELTING AT MORs

SUMMARY POINTS

1. Two-thirds of the Earth is resurfaced about every 100 million years. MOR spreading rates range from less than 10 mm yr$^{-1}$ to nearly 200 mm yr$^{-1}$.
2. The solidus is the transition from complete solid to partial melt. Melting proceeds from 0% at the solidus to 100% several hundred degrees higher at the liquidus.
3. Hotter mantle results in more melt production and thicker ocean crust.
4. More melt production results in shallower depth to ridges.
5. Water will substantially increase the amount of melting beneath MORs. Each 0.1% of water added to the mantle lowers the solidus by 150$^\circ$C to 250$^\circ$C.

Figure 1. Diagrams illustrating the melting mechanisms beneath ocean ridges. At any one pressure, the mantle melts over a temperature range of several hundred degrees. The boundary between melt absent and melt present is called the mantle solidus. As mantle ascends beneath the ocean ridge, it begins melting as the solidus is crossed, and melts progressively during further ascent. Thus, the mantle melts by pressure decrease rather than by temperature increase. Hot mantle crosses the solidus at greater depths, leading to a larger melting regime, greater extents of melting, and thicker crust than that produced by cold mantle. The numbers on the bottom diagrams correspond to the pressures where melting stops for the numbered flow lines on the upper diagrams. (Langmuir & Forsyth, 2007, Figure 1).
Figure 24
Figure 2. Plots of average compositions of ocean-ridge basalts (each point represents about 100 km of ridge length) vs. the average depth of the ridge. Na8.0 is the composition of basalt normalized to a constant MgO content of 8 wt.% to correct for shallow-level differentiation. High Na contents reflect small extents of melting, while lower Na contents reflect higher extents of melting. High extents of melting lead to low Na contents, greater crustal thickness, and shallower depths below sea level, consistent with a model of varying mantle temperature. (Langmuir & Forsyth, 2007, Figure 2).

Figure 25
Figure 3. Illustration of the effects of water on melting beneath ridges. Addition of water creates a deep ‘tail’ of low extents of melting, which contributes additional melt and causes greater crustal thickness. Although adding water causes the maximum extent and total amount of melt to increase, the average extent of melting across the whole depth of melting decreases because of the large, deep region of low-degree melts. (Langmuir & Forsyth, 2007, Figure 5a).
LITERATURE CITED
