Math + X Symposium on Seismology and Inverse Problems

Rice University · January 18–20, 2017

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BioScience Research Collaborative (BRC) Building, Room 280

Wednesday, January 18th

8:15am	**Maarten de Hoop**	*Welcome*
	Ned Thomas	*Opening Remarks*
8:30am	**Michel Campillo**	*Passive seismic monitoring: advancements and questions*
9:20am	**Gunther Uhlmann**	*Travel time tomography*
10:10am		BREAK
10:35am	**Victor Tsai**	*Thorny issues, idealistic solutions, and practical resolutions to seismological inverse problems*
11:25am	**Joonas Ilmavirta**	*Spectral rigidity of the round Earth*
12:15pm		LUNCH
1:15pm	**Robert van der Hilst**	*Imaging Earth's deep interior with scattered seismic waves*
2:05pm	**Matti Lassas**	*Inverse problems for linear and non-linear wave equations*
2:55pm		BREAK
3:20pm	**Paul Johnson**	*Predicting failure*
4:10pm	**Josselin Garnier**	*Correlation-based imaging in random media*

Thursday, January 19th

8:30am	**Bruce Buffett**	*Numerical models of planetary dynamos: challenges and opportunities*
9:20am	**Yousef Saad**	*Divide and conquer algorithms and software for large Hermitian eigenvalue problems*
10:10am		BREAK
10:35am	**Tom Dickens**	*An overview with full-waveform inversion in exploration geophysics*
11:25am	**Laurent Demanet**	*Bandwidth extension for wave-based imaging*
12:15pm		LUNCH

1:15pm	**Thorne Lay**	*Earthquake source rupture model inversions using multiple data types*
2:05pm	**Gen Nakamura**	*Identification of the elasticity tensor by boundary measurements*
2:55pm		——— BREAK ———
3:20pm	**Laura Pyrak-Nolte**	*Fracture geometry trajectories in state space: dynamic diagnostics*
4:10pm	**Bjorn Engquist**	*Full waveform inversion and optimal transport*

Friday, January 20th

8:30am	**Brian Kennett**	*Characterizing the multi-scale nature of the seismic lithosphere*
9:20am	**András Vasy**	*The local inverse problem for the geodesic X-ray transform on tensors and boundary rigidity*
10:10am		——— BREAK ———
10:35am	**Maureen Long**	*Imaging seismic anisotropy in the Earth's mantle: progress, prospects, and pitfalls*
11:25am	**Mikko Salo**	*Energy estimates for the geodesic X-ray transform*
12:15pm		——— LUNCH ———
1:15pm	**Alan Levander**	*Imaging lithospheric drips and delaminations with large seismic arrays*
2:05pm	**Allan Greenleaf**	*Bilinear operators and Fréchet differentiability in seismic inversion*
2:55pm		——— BREAK ———
3:20pm	**Lauri Oksanen**	*Computational methods for the inverse boundary value problem for the wave equation*
4:10pm	**Hanming Zhou**	*The X-ray transform with matrix weights, and applications*

Abstracts — Wednesday, January 18th

Michel Campillo (Université Grenoble-Alpes) *Passive seismic monitoring: advancements and questions*

Beyond traditional imaging, the seismic waves can be used to probe the mechanical properties of the rocks at depth. Particularly they allow for detecting temporal changes of these properties when the medium is subject to tectonic of external forcing. This can lead to an in situ dynamic characterization of the medium. Because of the high energy required to produce continuously waves that penetrate at depth, one has to rely onto ambient noise uncontrolled sources such as external forcing from hydro-meteorological origins or from human activity. Correlations of continuous ambient noise records allow retrieving virtual seismograms corresponding ideally to the Green function between the positions of the two receivers. Using the coda part (scattered waves) of the virtual seismograms makes possible to measure slight changes in the seismic velocity and in scattering cross sections. While the direct waves in the virtual seismograms are sensitive to the spatial distribution of the noise sources, the coda waves are more robust, leading in practice to stability sufficient for the monitoring of the actual changes. We present examples of response to impulsive changes like an earthquake or a mining blast. We illustrate the effect of hydro-meteorological forcing and of permanent periodic deformations associated with tides or thermo-elastic response to surface temperature. We discuss the main theoretical issues to be solved for further progresses of the approach.

Work with Qingyu Wang, Shujuan Mao, Gerritt Olivier, Florent Brenguier, Gregor Hillers, Laurent Stehly, Nikolai Shapiro, Robert van der Hilst.

Gunther Uhlmann (University of Washington) *Travel time tomography*

We will consider the inverse problem of determining the sound speed or index of refraction of a medium by measuring the travel times of waves going through the medium. This problem arises in global seismology in an attempt to determine the inner structure of the Earth by measuring travel times of seismic waves. It has also several applications in optics and medical imaging among others.

The problem can be recast as a geometric problem: Can one determine a Riemannian metric of a Riemannian manifold with boundary by measuring the distance function between boundary points? This is the boundary rigidity problem. We will also consider the problem of determining the metric from the scattering relation, the so-called lens rigidity problem. The linearization of these problems involve the integration of a tensor along geodesics, similar to the X-ray transform.

We will also describe some recent results, joint with Plamen Stefanov and András Vasy, on the partial data case, where you are making measurements on a subset of the boundary. No previous knowledge of Riemannian geometry will be assumed.

Victor Tsai (Caltech) *Thorny issues, idealistic solutions, and practical resolutions to seismological inverse problems*

Despite recent advances in seismological inverse theory and many large new datasets, robustly inferring the fine-scale structure of the Earth and dynamics of earthquakes remains not only challenging but essentially infeasible in detail. Primary issues include

(1) the computational cost of the forward problem being prohibitively expensive to evaluate for the wide range of possible realistic Earth structures,

(2) a lack of understanding of the physics of friction, thus not allowing dynamic rupture models to be ideally parameterized,

(3) inaccuracies in assumed seismometer instrumental response to Earth motions, primarily related to installation and microscale near-station structure, and

(4) the difficulty of performing quality control measures on very large volumes of data.

For these issues to be overcome, I indicate a need to

(1) incorporate dynamical and mechanical modeling into structural inversions to provide hypotheses to be discriminated between,

(2) prove that frictional models do not permit certain solutions and can be falsified,

(3) model the inelastic behavior of seismometer installations, and

(4) create deglitched data streams by incorporating peripheral information.

Since definitive solutions in these areas will no doubt take many years to become reality, in the meantime I suggest alternative paths forward for making robust forward progress that involve

(1) restricting models to strictly overdetermined parameterizations,

(2) incorporating more hypothesis testing with inversion results,

(3) utilizing alternative approximate theories to test for robustness of results, and

(4) being conservative with data quality metrics.

I provide examples of these practical approaches to challenging issues with seismological inverse problems.

Joonas Ilmavirta (University of Jyväskylä) *Spectral rigidity of the round Earth*

Reconstructing the interior structure of the Earth from only the spectrum of its free oscillations is impossible for several reasons. This kind of "spectral uniqueness" is beyond reach, but a weaker result known as "spectral rigidity" is not. We discuss what it means if a model of the Earth is spectrally rigid and we give the first spectral rigidity result for a simple radial model.

Matti Lassas (University of Helsinki) *Inverse problems for linear and non-linear wave equations*

We consider uniqueness and stability results for inverse problems for hyperbolic equations in an anisotropic medium. Our aim is to determine the Riemannian metric, associated to travel times of waves, inside a domain from the observations done on the boundary.

Typical inverse problems in anisotropic media are not uniquely solvable: A change of coordinates changes the equation but does not change the boundary data. This point of view makes it possible to consider both uniqueness results and construct counterexamples.

To prove uniqueness results, one may consider properties that are invariant in smooth diffeomorphisms and try to reconstruct those uniquely. For example, there is an underlying manifold structure that can be uniquely determined. Thus the inverse problem in a subset of the Euclidean space can be solved in two steps. The first one is to reformulate the problem in terms of manifolds and to reconstruct the underlying manifold structure from the boundary data. The second step is to find an embedding of the constructed manifold to the Euclidean space.

For inverse problems for the linear wave equation, we consider methods based on unique continuation, in particular the boundary control method. We present new uniqueness and stability results for these problems.

For non-linear wave equations we develop a new method that utilises the non-linearity as a tool in imaging. The main idea is to reduce the inverse boundary value problem to a passive imaging problem where one observes waves coming from the point sources that are inside the domain. The latter passive problem can be solved using methods of Riemannian and Lorentzian geometry.

Paul Johnson (Los Alamos) *Predicting failure*

Forecasting failure is the goal in diverse domains that include earthquake physics, materials science, nondestructive evaluation of materials and other engineering applications. Due to the highly complex physics of material failure, the goal appears out of reach; however, recent advances in instrumentation sensitivity, instrument density and data analysis show promise toward forecasting failure times. In this work we show that we can predict frictional failure times ('labquakes') in laboratory shear experiments. This advance is made possible by applying Random Forest (RF) machine learning to the continuous time series recorded by a single accelerometer listening to the experiment. The RF is trained applying a number of statistical data features over a time interval over which a number of labquakes occur. Remarkably, during testing we find that the RF predicts upcoming failure time immediately following a labquake, based only on a short time window of data—a 'now' prediction. The predicted time improves as failure is approached, as other data features add to prediction. The approach should be portable to most or all failure problems.

Joint work with Bertrand Rouet-Leduc, Claudia Hulbert, Nicholas Lubbers, and Kipton Barros.

Josselin Garnier (École Polytechnique) *Correlation-based imaging in random media*

Sensor array imaging in a randomly scattering medium is usually limited because coherent signals recorded by the array and coming from a reflector to be imaged are weak and dominated by incoherent signals coming from multiple scattering by the medium. Stochastic and multiscale analysis allows to understand the direct problem and helps solving the inverse problem. We will see in this talk how correlation-based imaging techniques can mitigate or even sometimes benefit from the multiple scattering of waves. Applications to seismic interferometry and virtual source imaging will be discussed, as well as applications to non-destructive testing and intensity correlation imaging in optics.

Abstracts — Thursday, January 19th

Bruce Buffett (UC Berkeley) *Numerical models of planetary dynamos: challenges and opportunities*

Planetary magnetic fields are sustained over geological timescales by fluid motions in the planetary interiors. Models first achieved self-sustaining magnetic fields about twenty years ago, and the subsequent time has brought a steady pace of improvement. Many important aspects of planetary magnetic fields can now be reproduced, including the dipole dominance and the erratic occurrence of polarity reversals. However, it is generally acknowledged that these models are unrealistic in many respects. The main problem is the use of large diffusivities to compensate for limited spatial resolution. As a consequence, the nature of the dynamics is altered and the potential to address scientific questions is limited. Two challenges stand in the way of major advances. The first involves the numerical method, which must be designed to exploit the next generation of supercomputers. The second challenge involves the need for better models to deal with the effects of unresolved turbulence because resolved calculations at realistic conditions are unlikely in the foreseeable. Both of these challenges are the focus of recent efforts. Here I report on a recent benchmarking exercise to assess the most promising computational method for efficient computations on $O(10^6)$ processor cores. I also describe recent efforts to develop subgrid-scale models that can account for strong, anisotropic structure in the small-scale flow due to the combined influences of planetary rotation and an internal magnetic field. Progress on both fronts would bring dramatic improvements in our ability to interpret planetary magnetic fields and even forecast the behavior of Earth's magnetic field.

Yousef Saad (University of Minnesota) *Divide and conquer algorithms and software for large Hermitian eigenvalue problems*

Divide-and-conquer paradigms can lead to efficient and flexible techniques for solving large Hermitian eigenvalue problems. This talk will discuss how these techniques can be put to work to implement 'spectrum slicing' strategies, i.e., strategies that extract slices of the spectrum independently. The presentation will begin with an overview of polynomial filtering, a general approach that can be quite efficient in the situation where the matrix-vector product operation is inexpensive and when a large number of eigenvalues is sought. We will present a few techniques based on the Lanczos algorithm with and without restarts, as well as subspace iteration. An alternative to polynomial filtering that is generating a growing interest is a class of methods that rely on rational functions. Good representatives of this general approach are the FEAST eigensolver and the Sakurai-Sugiura algorithm. We will argue that the standard Cauchy integral-based approach can be substantially improved upon — especially when iterative solvers are involved. Finally, the talk will discuss our recently released code named EVSL (for eigenvalues slicing library) that implements these ideas.

Tom Dickens (ExxonMobil) *An overview with full-waveform inversion in exploration geophysics*

We present an overview of full-waveform inversion (FWI), focusing on its use in seismic exploration. We consider the typical data acquisition scenarios used by industry, and discuss the progression of velocity model building and imaging techniques that have been employed to extract increasing amounts of information from seismic data. FWI has the potential to make the most complete use of seismic data, providing information ranging from improved imaging velocities to maps of rock and fluid properties on sub-wavelength scales. While the basic mathematics of FWI has been known for decades, only in the last ten years or so have algorithm and compute-power advances made practical application to seismic problems possible. A number of these advances are discussed, following a brief discussion of the basic mathematical techniques used in FWI. Several synthetic examples illustrating the performance of FWI in ideal conditions are then shown. Finally, we look at some examples of the challenges that must be overcome to make FWI usable in real-world scenarios, and present examples of research aimed at solving these problems.

Laurent Demanet (MIT) *Bandwidth extension for wave-based imaging*

This talk considers the basic question of frequency extrapolation of bandlimited recordings of scattered waves. I will review recent progress on the mathematical aspects of this question, which are tied to the notion of super-resolution. I will also discuss two methods that were shown to give meaningful results for seismic imaging:

(i) a model reduction approach, where the phases of atomic seismic events are estimated by tracking, and

(ii) a model extension approach, based on TV-regularized least-squares inversion of the extended Born modeling operator.

Both methods are meaningful in the sense that they can help bootstrap the frequency sweeps for full waveform inversion. Joint work with Yunyue Elita Li.

Thorne Lay (UC Santa Cruz) *Earthquake source rupture model inversions using multiple data types*

Recent large earthquakes around the world are commonly recorded by a mixture of seismic, geodetic and tsunami observations. These diverse data types can be brought to bear on constructing unified representations of the source process compatible with all of the observations. Technical challenges in doing so include the computation of self-consistent Green functions for the diverse data types using adequate theoretical representation of the signal excitation and propagation, and combination of the diverse data types into joint inversion or iterative modeling schemes that respect the relative information about the source provided by different signals. Understanding the intrinsic sensitivity of each data type to the source characteristics is critical for the joint analysis. Examples of the challenges and applications of such joint data analyses in the context of linear least squares inversions and iterative modeling with non-linear calculations (necessary for some tsunami signals) will be discussed using recent great earthquake examples.

Gen Nakamura (Hokkaido University) *Identification of the elasticity tensor by boundary measurements*

The uniqueness of the Calderon problem for three-dimensional elasticity has not been solved even for isotropic elasticity. So far there is a partial result by Nakamura-Uhlmann and Eskin-Ralston for isotropic elasticity. In this talk I will consider the case that the elasticity tensor is piecewise constant or piecewise analytic. The anisotropy of elasticity tensor can be allowed in some cases.

This study stemmed from joint work with C. Carsteas (National Taiwan University).

Bjorn Engquist (UT Austin) *Full waveform inversion and optimal transport*

Full Waveform Inversion as a technique for inverse problems in seismology requires a measure of mismatch between computed seismic waves and measured data. We will briefly discuss such measures in general and then focus on the application of optimal transport and the Wasserstein distance to this problem. Some advantages of the Wasserstein distance over more traditional measures will be presented in analytical results as well as by numerical examples.

Abstracts — Friday, January 20th

Brian Kennett (Australian National University) *Characterizing the multi-scale nature of the seismic lithosphere*

Understanding the complex heterogeneity of the continental lithosphere involves a wide variety of spatial scales and the synthesis of multiple classes of information. Seismic surface waves provide the main constraints on broad-scale structure. Information on finer scale structures comes through body wave studies, including detailed seismic tomography and P wave reflectivity extracted from stacked autocorrelograms of continuous component records.

There are complex interactions between the different scale and components of heterogeneity. Assessment of multi-scale models across a broad band of frequencies requires extensive numerical computation, but can only look at a limited number of realizations of stochastic processes. Nevertheless progress can be made on finding models that provide a good representation of the character of seismic observations across the Australian continent. With the inclusion of deterministic large-scale structure and realistic medium-scale stochastic features there is not a need for strong fine-scale variations, but hints that there is vertical variability in the character of heterogeneity.

András Vasy (Stanford) *The local inverse problem for the geodesic X-ray transform on tensors and boundary rigidity*

In this talk, based on joint work with Plamen Stefanov and Gunther Uhlmann, I discuss the geodesic X-ray transform on a Riemannian manifold with boundary. The geodesic X-ray transform on functions associates to a function its integral along geodesic curves, so for instance in domains in Euclidean space along straight lines. The X-ray transform on symmetric tensors is similar, but one integrates the tensor contracted with the tangent vector of the geodesics. I will explain how, under a convexity assumption on the boundary, one can invert the local geodesic X-ray transform on functions, i.e. determine the function from its X-ray transform, in a stable manner. I will also explain how the analogous result can be achieved on one forms and 2-tensors up to the natural obstacle, namely potential tensors (forms which are differentials of functions, respectively tensors which are symmetric gradients of one-forms).

Here the local transform means that one would like to recover a function (or tensor) in a suitable neighborhood of a point on the boundary of the manifold given its integral along geodesic segments that stay in this n eighborh ood (i.e. with both endpoints on the boundary of the manifold). Our method relies on microlocal analysis, in a form that was introduced by Melrose.

I will then also explain how, under the assumption of the existence of a strictly convex family of hypersurfaces foliating the manifold, this gives immediately the solution of the global inverse problem by a stable 'layer stripping' type construction. Finally, I will discuss the relationship with, and implications for, the boundary rigidity problem, i.e. determining a Riemannian metric from the restriction of its distance function to the boundary.

Maureen Long (Yale) *Imaging seismic anisotropy in the Earth's mantle: progress, prospects, and pitfalls*

Because of the causative link between deformation and seismic anisotropy in the Earth's mantle, observations of seismic anisotropy can yield important constraints on the present-day pattern of mantle flow and on deformation associated with past tectonic events. Despite its importance, seismic anisotropy is often difficult to constrain, particularly in the context of tomographic imaging of mantle structure. This difficulty stems from several factors, including the strongly nonlinear relationship between anisotropy parameters and observables such as shear wave polarizations and splitting delay times, the large number of parameters needed to describe anisotropy, and the practical challenges inherent in the measurement of shear wave splitting in noisy data. In this presentation I will discuss the challenges associated with imaging anisotropy in the Earth's mantle, along with recent progress and challenges for the future. In particular, I will discuss strategies for the application of finite-frequency shear wave splitting intensity tomography, the implementation of model space search approaches for constraining anisotropic parameters, and strategies for integrating constraints from geodynamical modeling and mineral physics into the interpretation of seismic anisotropy measurements.

Mikko Salo (University of Jyväskylä) *Energy estimates for the geodesic X-ray transform*

The Euclidean X-ray transform, which encodes the integrals of a function over straight lines, is a classical topic (going back to J. Radon in 1917) and forms the basis of imaging methods such as X-ray computed tomography. The geodesic X-ray transform encodes the integrals of a function over more general families of curves, such as the geodesics of a sound speed or Riemannian metric. It arises in seismic imaging as the linearization of the travel time tomography problem, and is also related to Electrical Resistivity Tomography (the Calderón problem) and inverse spectral problems.

There has been considerable recent progress in understanding the mathematical properties of the geodesic X-ray transform, based on several different methods. In this talk we will review results related to energy estimates for the underlying transport PDE, partly based on joint works with G. Paternain (Cambridge) and G. Uhlmann (Washington).

Alan Levander (Rice) *Imaging lithospheric drips and delaminations with large seismic arrays*

Ocean lithosphere is created at mid-ocean ridges and recycled to the mantle with a time constant of about $\sim$150 Myr. Small parts of the mantle under the stable continental interiors have been destabilized enough for recycling, but most has persisted intact for ≥ 2 Gyr. Roughly 15% of the Earth's surface is characterized as diffuse plate boundary zones, two thirds of which encompasses the tectonically active continental mountain belts and all three of the Earth's large orogenic plateaus. Most orogenic belts have a thin mantle lithosphere compared to the oceans or continental interiors, suggesting that their mantle lithosphere has been recycled to the mantle during the orogenic cycle.

The loss of lithospheric mantle from beneath orogenic belts and plateaus has been ascribed to several possible causes: initiation of Rayleigh-Taylor instabilities by surface or intra-crustal loads applied to otherwise stable, density-stratified fluids (Houseman and Molnar, 1981; 2001), delamination of the mantle lithosphere (Byrd, 1979), ablative or viscous entrainment during subduction (Tao and O'Connell, 1992; Lenardic, 1993), and edge convection (e.g., Van Wijk et al., 2010). Strictly speaking these mechanisms are not part of classic plate tectonic theory. To initiate a downwelling requires a combination of an external trigger, and other mediating physical factors, such as lithospheric weakening through hydration, application of a surface load, development of dense crustal rocks through melting, or advantageous geometry of a nearby subduction zone.

Lithosphere removal as delamination events was originally suggested from geochemical evidence and other geologic data. The development and deployment of large portable broadband seismograph arrays has resulted in fairly detailed images of drip and delamination-like features under most orogenic belts including the western Mediterranean (Seber et al., 1996; Levander et al, 2014; Bezada et al., 2014), and Alpine system (Lippitsch et al., 2003; Fox et al., 2015), the Tibetan Plateau (Chen et al, 2017), the Venezuelan Cordillera (Levander et al., 2014), and the North and South American Cordillera (Schmandt and Humphreys, 2010; Zandt et al, 2004, Hales et al, 2005; Gao et al, 2004; Levander et al., 2011; Beck et al, 2015). Detailed imaging of these various drips suggest that each of the conceptual models is operative somewhere on the globe, but usually in combination with one or more of the others.

In the western U.S., six upper mantle, positive velocity anomalies seen in USArray regional tomography images are interpreted as downwellings (Schmandt and Humphreys, 2010). In map view these drips occupy between 5% and 10% of the surface area of the western US orogenic belt. Sinking at plate tectonic rates, descending drips can be positively identified for 5-10 Myr. This suggests that during a 100 Myr orogenic lifespan, most of the lithospheric mantle incorporated into an orogenic belt can be recycled to the deeper mantle.

Allan Greenleaf (Rochester) *Bilinear operators and Fréchet differentiability in seismic inversion*

High frequency linearized seismic imaging uses techniques from microlocal analysis to understand how singularities in the sound speed are transformed into the data from seismic experiments. In particular, it shows how multipathing combines with the type of seismic data (single shot, marine,…) to result in artifacts in backprojected reconstructions. However, until recently the theoretical basis for this, namely the Fréchet differentiability of the map taking the sound speed to the data, was not known. In 2013, Kirsch and Rieder proved this for certain pairs of function spaces. After giving an incomplete survey of the area, I will describe preliminary work on trying to establish similar results for function spaces adapted to the degeneracies produced by multipathing.

This is joint work with Margaret Cheney, Raluca Felea, Romina Gaburro and Cliff Nolan.

Lauri Oksanen (University College London) *Computational methods for the inverse boundary value problem for the wave equation*

The inverse boundary value problem for the acoustic wave equation is one of the simplest coefficient determination problems where the data is generated by a hyperbolic partial differential equation. This problem is known to have a unique solution, however, there are not many attempts to develop computational methods that are guaranteed to converge to the unique solution when no initial guess for the speed of sound is available. The techniques that do not depend on a good initial guess lead to unstable computations. We study these instabilities and their regularization, and give computational examples.

The talk is based on a joint work with M. de Hoop and P. Kepley.

Hanming Zhou (Cambridge) *The X-ray transform with matrix weights, and applications*

We consider the inverse problem of determining (vector valued) functions and 1-forms from their integrals with (matrix) weights along geodesics on a manifold with strictly convex boundary. We show unique determination, modulo some natural obstructions, for manifolds admitting strictly convex functions in dimension three and higher and for generic ones in arbitrary dimension. Then we apply our results to study some non-linear inverse problems arising naturally from geophysics and mathematical physics. Part of the talk is based on a joint work with G. P. Paternain, M. Salo and G. Uhlmann.